# FFT-upsampling for Smoke Animation

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**Abstract**

*Fluid details are visually interesting but computationally challenging in animation. This paper introduces a simple frequency domain upsampling method to produce small scale details for smoke animation. Our method is based on a fluid solver combining fine grid advection with coarse grid projection. Different from previous work, we use a novel FFT-upsampling operator to reconstruct a fine velocity field from the coarse divergence-free velocity field. The upsampled velocity field is kept divergence-free without solving Poisson equation on the fine grid. We provide several examples to demonstrate that our method can significantly reduce the computational cost for simulation while still producing compelling high-frequency details for smoke animation.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation

## 1. Introduction

Small scale fluid details are visually interesting but challenging to simulate with grid-based fluid animation techniques. The quality of simulation is tightly related to the size of the grid, which was limited by the amount of computational resource available. In this paper, we present a fast method for simulating smoke with small scale details by a combined fine-coarse grid Navier-Stokes solver, using a novel Fast Fourier Transform (FFT) upsampling operator as our core tool.

Lentine et al. [LZF10] proposed a method to accelerate projection step on fine-grid simulation for incompressible flow. Our work is inspired by their research and we adopt a similar framework combining fine and coarse grids to accelerate projection. The main difference is that, by taking advantage of the mass-conserving property of our novel FFT-upsampling operator, we do not solve Poisson equation on the fine grid, and only solve one Poisson equation on the coarse grid in each time step.

## 2. FFT-upsampling for Reconstructing Fine Velocity Field

Inspired by the zero-padding(ZP) technique from signal processing field [CdV92], we construct an upsampling operator by defining a zero-padding interpolator on frequency domain using Fast Fourier Transform (FFT).

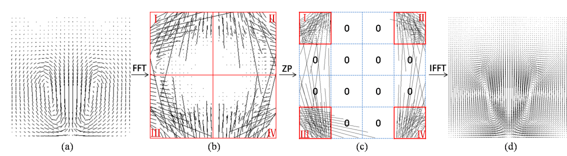


**Figure 1:** *Illustration of FFT-upsampling operator. (a) Coarse flow, (b) Fourier domain of coarse flow, (c) Fourier domain with zero-padding for upsampled flow with scale 2, (d) Upsampled flow with scale 2.*

As illustrated in Figure 1, we first convert the velocity field from spatial domain (Figure 1(a)) to frequency domain (Figure 1(b)) using FFT. Then we do zero-padding for the velocity field in frequency domain (Figure 1(c)) using a scale k (k=2,3,4..., here the Figure shows scale k=2). Finally, we obtain the interpolated velocity field in spatial domain by doing an inverse FFT transform, and get an upsampled result (Figure 1(d)).

Since this kind of zero-padding interpolation technique can provide near-perfect resampling for signals [CdV92], our FFT-upsampling operator can interpolate fluid details from coarse grid to fine grid and achieve a near-perfect reconstruction result. The FFT-upsampling operator do not add new high frequency details into the velocity field, but rather recover high frequency details in the velocity field as much as possible to the limit of Nyquist sampling frequency.

## 3. Results

We run all simulations and rendering on an Intel Xeon 2.80GHz CPU with 6GB of memory. First, we give a 2D example of smoke to demonstrate our FFT-upsampling method is effective, as shown in Figure 2. Figure 2(a) is a base simulation on a $32 \times 64$ coarse grid. Figure 2(b) is a simulation on a $128 \times 256$ grid and a $32 \times 64$ coarse grid using linear interpolation. Figure 2(c) shows our method on a $128 \times 256$ grid which is reconstructed from a $32 \times 64$ coarse grid. Figure 2(d) is a base simulation on a $128 \times 256$ grid. Note that the result of our method in Figure 2(c) is basically same in structure with that of the fine grid in Figure 2(d), while the linear interpolated result in Figure 2(b) is structurally similar to the base simulation on a coarse $32 \times 64$ grid in Figure 2(a). Obviously, our result in Figure 2(c) has more details than the linear interpolated result in Figure 2(b). More importantly, the result upsampled with linear interpolation is not divergence-free in refined velocity field, while our FFT-upsampling method still maintain the divergence-free condition in reconstructed field.

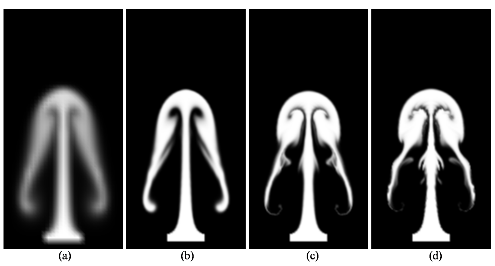


**Figure 2:** *2D rising smoke.*

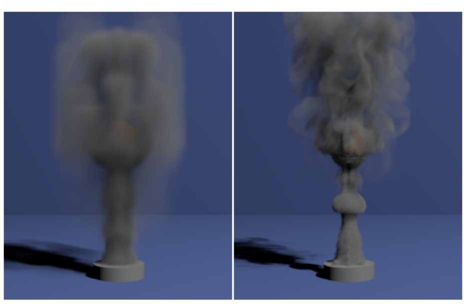

**Figure 3:** *3D uprising smoke with a static sphere. Left is a base simulation on a $32 \times 64 \times 32$ grid, right is our method using a $128 \times 256 \times 128$ grid and a $32 \times 64 \times 32$ coarse grid.*

As in Figure 3, we show an uprising 3D smoke example interacting with a static sphere. This example is carried out with 4 times scale of FFT-upsampling to illustrate that our method achieved much higher simulation quality than traditional solver. More test cases of FFT-upsampling are showed in Figure 4.

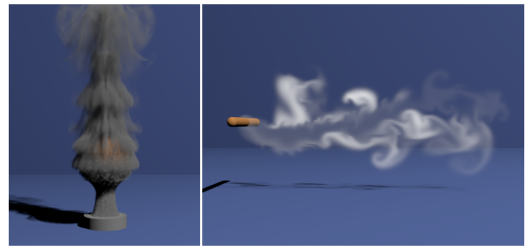

**Figure 4:** *More test cases of FFT-upsampling.*

The timings for our simulations are shown in Table 1. We compare our method with a base simulation on the coarse grid and a base simulation on the fine grid. It shows that our method did not bring much burden to the original coarse-grid simulation, while spend dramatically less time than the fine-grid ones. The table shows our method runs approximately 151 times faster on the high-resolution 3D smoke example.

**Table 1:** *Timing information. In 2D Smoke, the coarse grid is $32 \times 64$, and the fine grid is $128 \times 256$. In 3D Smoke, the coarse grid is $32 \times 64 \times 32$, and the fine grid is $128 \times 256 \times 128$. All timings are sec-per-frame.*

| Example | Coarse | Fine | Our method | Speedup |
|---|---|---|---|---|
| 2D Smoke | 0.015 | 0.188 | 0.016 | 11.75 |
| 3D Smoke | 1.716 | 851.29 | 5.601 | 151.94 |

## 4. Conclusion

We proposed a novel Fourier domain upsampling method for smoke animation, using zero-padding in frequency domain. This upsampling technique can preserve the incompressibility of velocity field when mapping from coarse grid to fine grid, without solving Poisson equation on the fine grid. Consequently, our method can provide convincing smoke details while effectively reduces the amount of computational time by one to two order of magnitude comparing with traditional simulator on fine grid. Besides, our technique is very simple to implement.

### Acknowledgement

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### References

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[LZF10] Lentine M., Zheng W., Fedkiw R.: A novel algorithm for incompressible flow using only a coarse grid projection. *ACM Trans. Graph. (Proc. SIGGRAPH) 29* (July 2010), 114:1–114:9. 1